Adjustment Computation

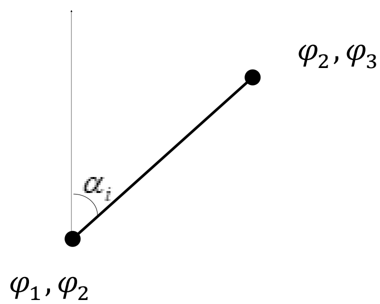
The fundamental principle can be expressed as follows:

"In observations of equal precision, the most probable values of the observed quantities are those that render the sum of the squares of the residual errors a minimum."

There are mainly two methods to perform the Least Square Adjustment

- Using Observation Equations
- Using Conditional Equations for Triangulation computations use observation equations.

Observation Equations for Triangulation Computations



$$\alpha_1 = f_1(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$
$$\alpha_2 = f_2(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$
$$\vdots$$
$$\alpha_n = f_n(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$

In a set of observations.

If N >U adjustment is possible

Where, N= Number of observations

U= Number of unknowns

$$\alpha_1^{'} + v_1 = f_1(\varphi_1^0 + d\varphi_1, \lambda_1^0 + d\lambda_1, \varphi_2^0 + d\varphi_2, \lambda_2^0 + d\lambda_2)$$

$$\alpha_1^{'} + v_1 = f_1(\varphi_1^0, \lambda_1^0, \varphi_2^0, \lambda_2^0) + \left[\frac{\partial f_1}{\partial \varphi_1} d\varphi_1\right] + \left[\frac{\partial f_1}{\partial \lambda_1} d\lambda_1\right] + \left[\frac{\partial f_2}{\partial \varphi_2} d\varphi_2\right] + \left[\frac{\partial f_2}{\partial \lambda_2} d\lambda_2\right]$$

$$v_1 = \left(\alpha_1^0 - \alpha_1^{'}\right) + a_1 d\varphi_1 + b_1 d\lambda_1 + c_1 d\lambda_2 + d_1 d\lambda_2$$
$$\vdots$$
$$v_n = \left(\alpha_n^0 - \alpha_n^{'}\right) + a_n d\varphi_1 + b_n d\lambda_1 + c_n d\lambda_2 + d_n d\lambda_2$$

Observation Equation for the Azimuth AB,

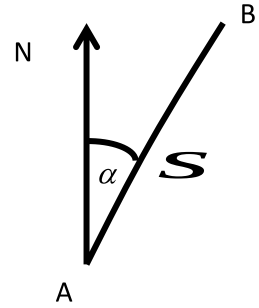


Figure: Azimuth of a line.

$$v = \left(\alpha^0 - \alpha'\right) + d\alpha_{ABt}$$

where, $d\alpha_{ABt} = \frac{1}{AB}\left[M_A \sin\alpha_{AB}.d\varphi_A + M_B \sin\alpha_{BA}.d\varphi_B - N_B \cos\varphi_B \cos\alpha_{BA}(d\lambda_B - d\lambda_A)\right]$

In case that the observed quantities are included angles

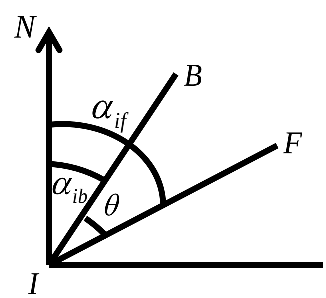


Figure: Azimuth of a line using include angles.

$$\theta = \alpha_{if} - \alpha_{ib}$$

We know that,

$$v_{if} = \left(\alpha_{if}^0 - \alpha_{if}'\right) + d\alpha_{ift}$$

$$v_{ib} = \left(\alpha_{ib}^0 - \alpha_{ib}'\right) + d\alpha_{ibt}$$

i-Instrument station

b-Backward Station

f-Forward station

So now the observation equation for included angle,

$$v_{if} - v_{ib} = (\alpha_{if}^0 - \alpha_{ib}^0) - (\alpha'_{if} - \alpha'_{ib}) + (d\alpha_{ift} - d\alpha_{ibt})$$

$$v_\theta = (\theta^0 - \theta') + d\alpha_\theta$$

where

$$d\alpha_\theta = d\alpha_{ift} - d\alpha_{ibt}$$
$$= \frac{1}{IF}\left[M_i \sin\alpha_{if}.d\varphi_i + M_f \sin\alpha_{fi}.d\varphi_f - N_f \cos\varphi_f \cos\alpha_{fi}(d\lambda_f - d\lambda_i)\right] -$$
$$\frac{1}{IB}\left[M_i \sin\alpha_{ib}.d\varphi_i + M_b \sin\alpha_{bi}.d\varphi_b - N_b \cos\varphi_b \cos\alpha_{bi}(d\lambda_b - d\lambda_i)\right]$$

where

$d\alpha_t$= Error of the azimuth difference.
M_i = Meridian radius of curvature at the instrument station.
M_f= Meridian radius of curvature at the forward station.
M_b= Meridian radius of curvature at the back station.
N_i= Prime vertical radius of curvature at the instrument station.
N_f= Prime vertical radius of curvature at the forward station.
N_b= Prime vertical radius of curvature at the back station.
$d\varphi$, $d\lambda$= Error of latitude and longitude.
IB= Length of back sight.
IF = Length of fore sight.
α = Azimuth of the station

Using above observation equation calculate the Jacobean and f matrix for least square adjustment

$$\begin{bmatrix} v_1 \\ \vdots \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} a_1 \ b_1 \ c_1 \ d_1 \ \cdots \\ \vdots \\ \vdots \\ a_1 \ b_1 \ c_1 \ d_1 \ \cdots \end{bmatrix} \begin{bmatrix} d\varphi_1 \\ d\lambda_1 \\ d\varphi_2 \\ d\lambda_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ \vdots \\ \vdots \\ f_n \end{bmatrix}$$

$$V = B\Delta + F$$

Where $v_1, v_2, \cdots v_n$ Residuals

$\alpha_1^{'}$ – Observed value

α_1^0 – Computed value

$d\lambda_1, d\varphi_1, d\lambda_2, d\varphi_2$ - Correction for unknowns

$\varphi_1, \lambda_1, \varphi_2, \lambda_2$ -unknowns

according the concept of least square adjustment

The sum of the square of the residuals should be minimum ($\phi = \nu^T w \nu$);

i.e., $\frac{d\phi}{d\Delta} = 0$

Where w =weight matrix

So that $\frac{d\phi}{d\Delta} = 2 f^T WB + 2\Delta^T (B^T WB) = 0$

Therefore $(B^T WB)\Delta = B^T W(-f)$

Let $N = B^T WB$

$T = B^T W(-f)$

$N\Delta = T$

$\Delta = N^{-1} T$

$\Delta = (B^T WB)^{-1} B^T WB$

(References: Prasanna H.M.I (2014) Geodetic Computations on Triangulation)